Path-complete Lyapunov techniques

And open problems

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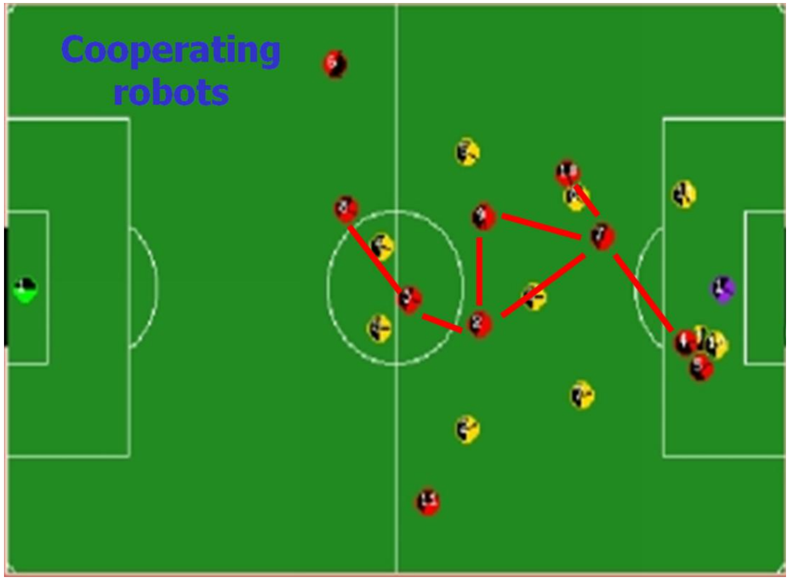
SAMSA 2025
Warsaw

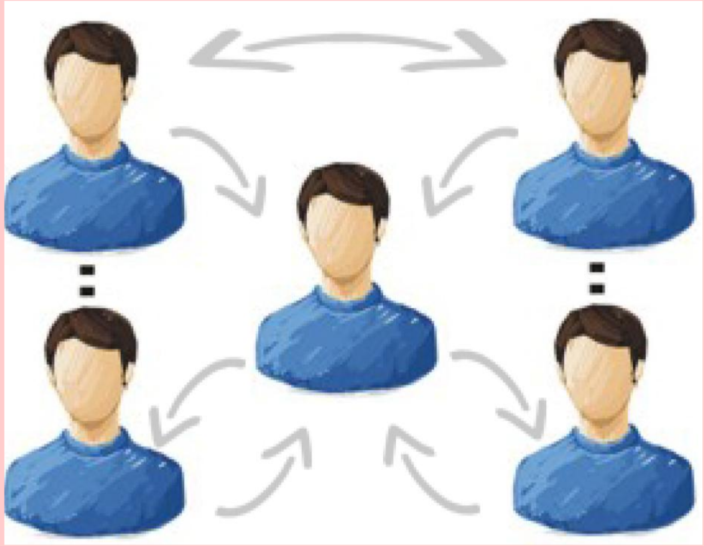
UCLouvain icteam European Research Council Established by the European Commission

Switching systems

Consensus/gossip algorithm



Recommender systems



Switching systems

Consensus/gossip algorithm

Outline

- Switching systems
- Path-complete methods for switching systems stability
- Further results
- SAMSA? ‘Series, Automata, Matrices, **Symbolic dynamics**, and their Applications’
- Conclusion and open questions

Outline

- **Switching systems**

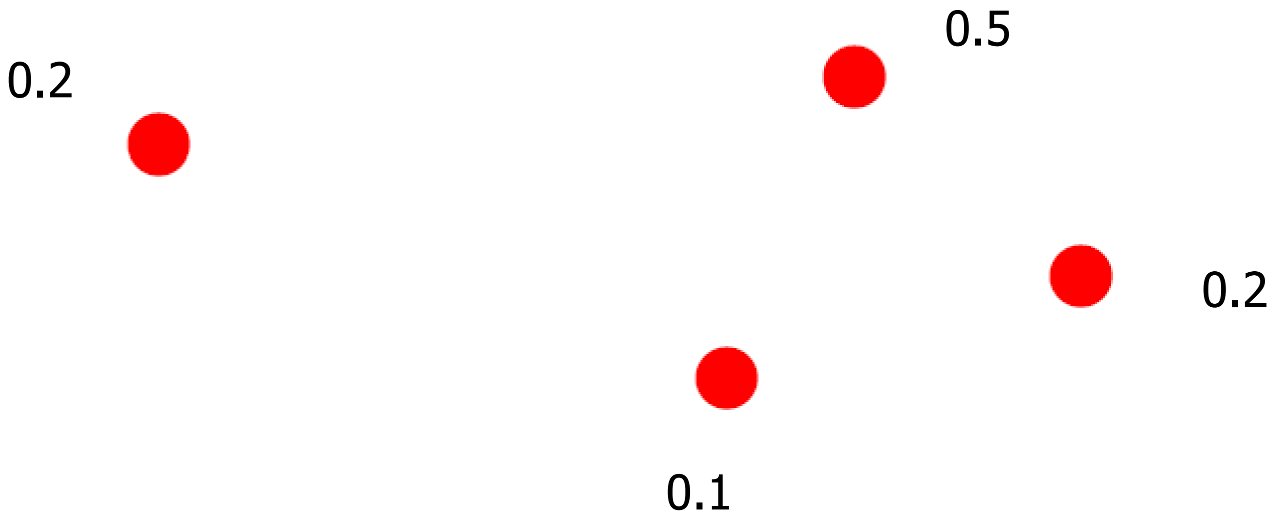
- Path-complete methods for switching systems stability

- Further results

- SAMSA? ‘**Series**, Automata, **Matrices**, Symbolic dynamics, and their **Applications**’

- Conclusion and open questions

Gossip algorithms



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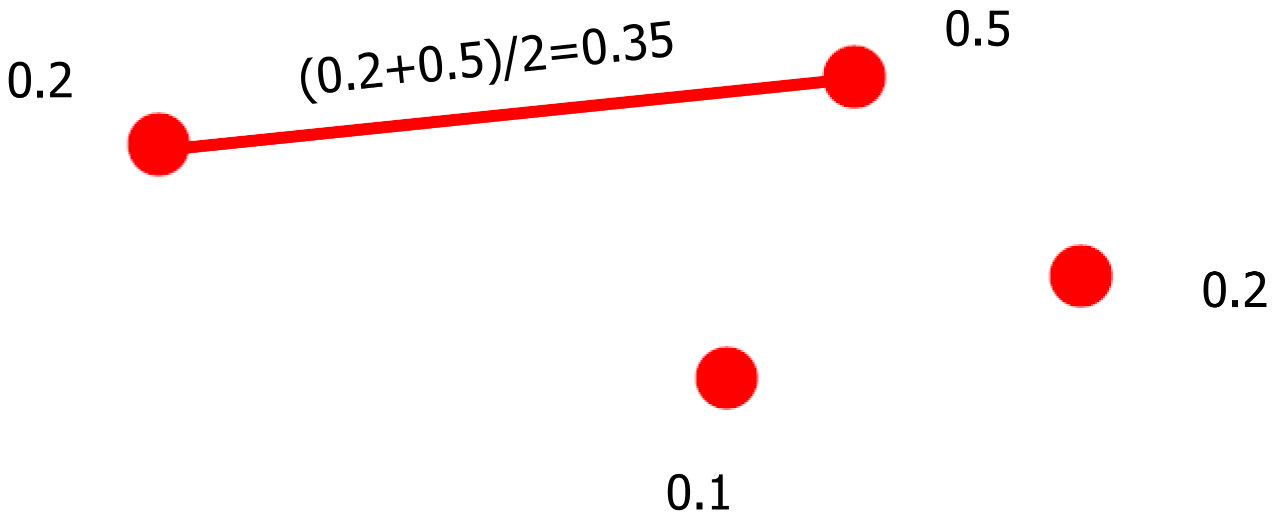
Gossip algorithms



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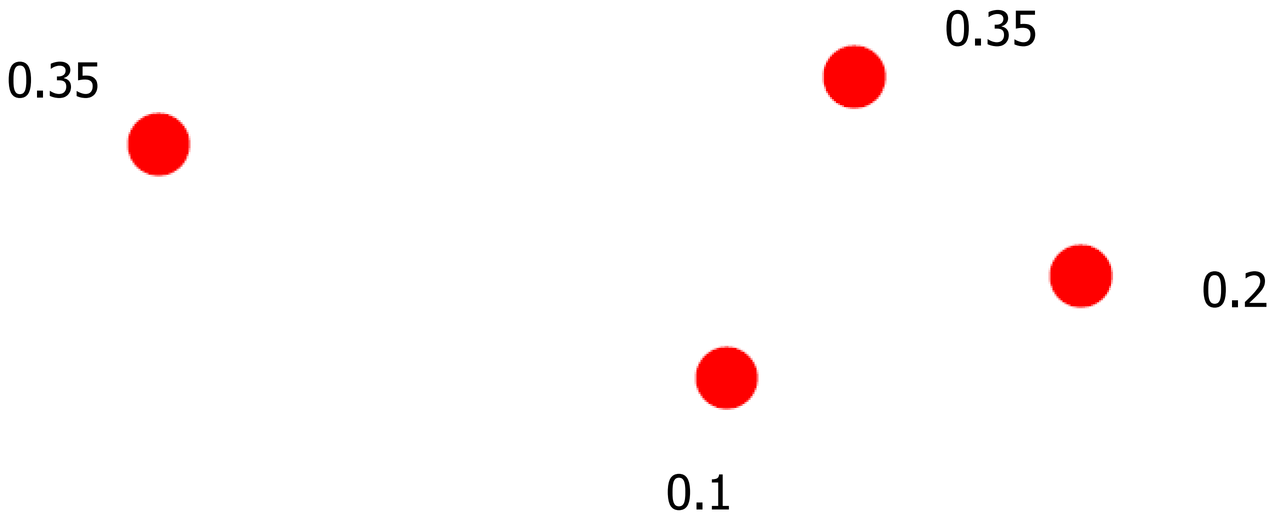

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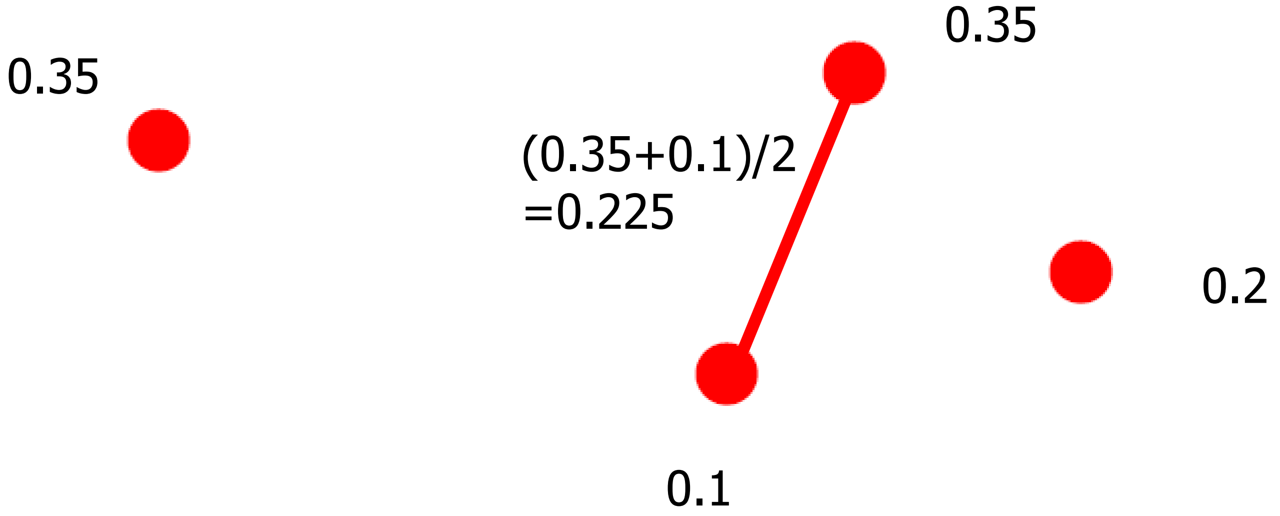

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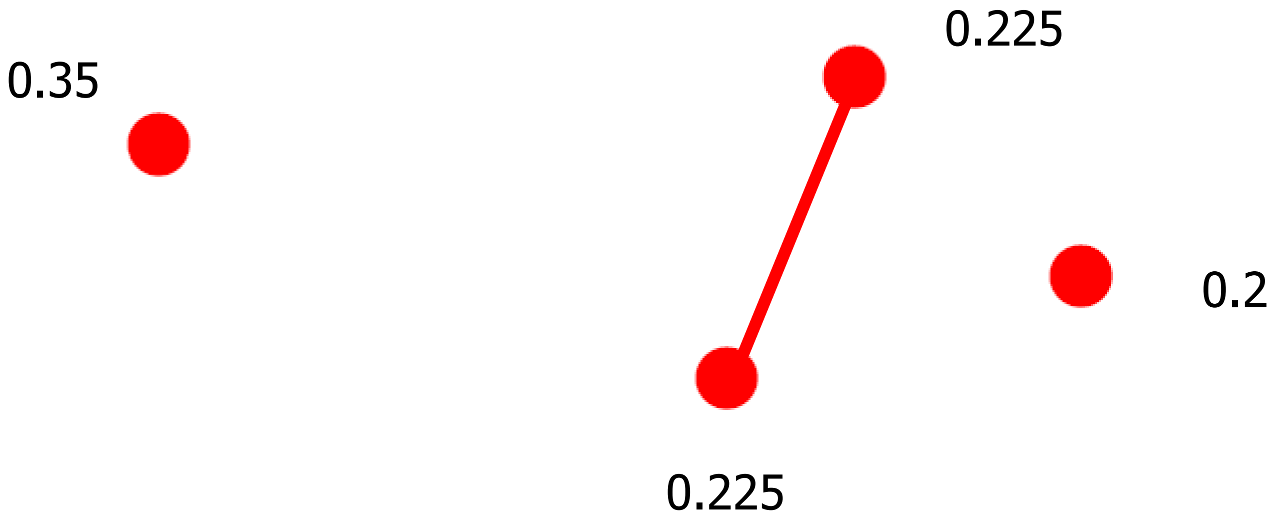
Gossip algorithms



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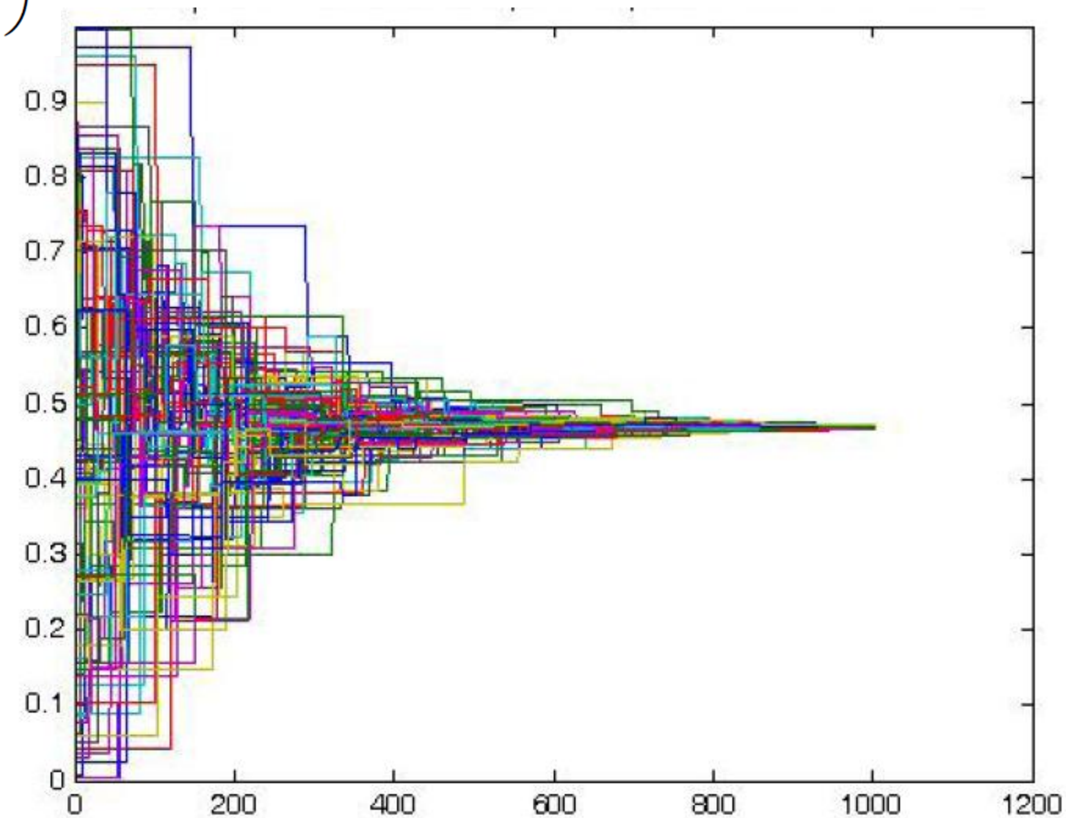
Gossip algorithms

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \leftarrow \begin{pmatrix} 0.5 & 0.5 & . & 0 \\ 0.5 & 0.5 & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$x(t+1) = A(t)\,x(t)$$



Switching systems

$$x(t+1) = A_0 x(t)$$

or

$$x(t+1) = A_1 x(t)$$

Point-to-point Given x_0 and x_*, is there a product (say, A_0 A_0 A_1 A_0 … A_1) for which $x_*=A_0$ A_0 A_1 A_0 … A_1 x_0?

Mortality Is there a product that gives the zero matrix?

Boundedness Is the set of all products $\{A_0, A_1, A_0A_0, A_0A_1, \ldots\}$ bounded?

Switching systems

$$x(t+1) = A_0 x(t)$$

or

$$x(t+1) = A_1 x(t)$$

Global convergence to the origin Do all products of the type $A_0\ A_0\ A_1\ A_0 \ldots A_1$ converge to zero?

The spectral radius of a matrix A controls the growth or decay of powers of A

$$\rho(A) = \lim_{t \to \infty} ||A^t||^{1/t}$$

The powers of A converge to zero iff $\rho(A) < 1$

The joint spectral radius of a set of matrices Σ is given by

$$\rho(\Sigma) = \lim_{t \to \infty} \max_{A_i \in \Sigma} ||A_1 A_2 \ldots A_t||^{1/t}$$

All products of matrices in Σ converge to zero iff $\rho(\Sigma) < 1$

[Rota, Strang, 1960]

Outline

- Switching systems
- **Path-complete methods for switching systems stability**
- Further results
- SAMSA
  Series, **Automata**, Matrices, Symbolic dynamics, and their Applications
- Conclusion and open questions

Switching systems stability (a.k.a. JSR computation)

The CQLF method (Common Quadratic Lyapunov Function)

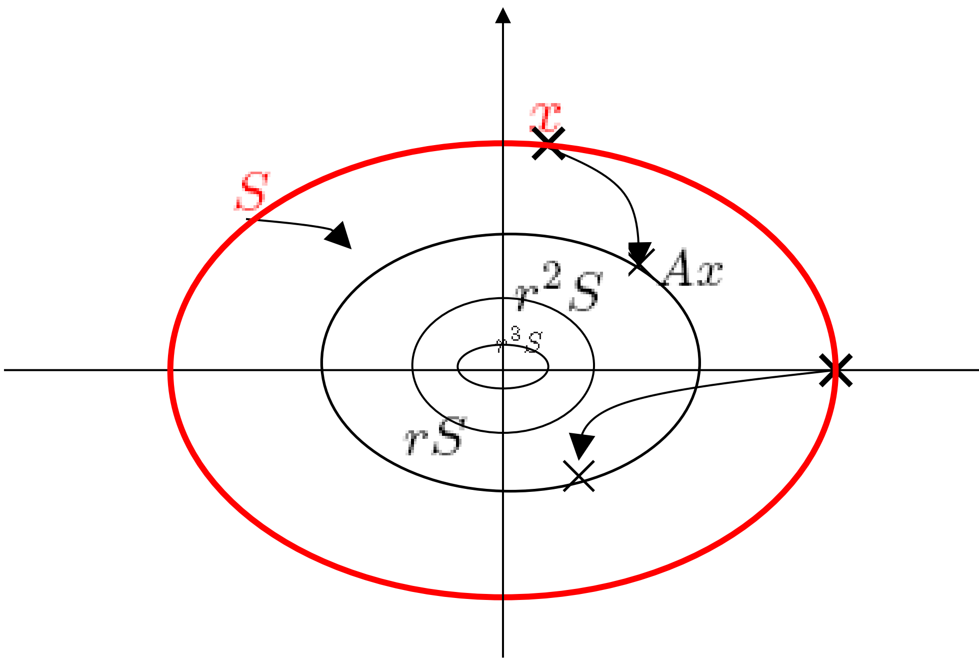
$$\begin{array}{llll} \inf_{r\in\mathbb{R}^+} & & r & \\ \text{s.t.} & & & \\ A^TPA & \preceq & r^2P, & \forall A\in\Sigma \\ P & \succeq & 0. & \end{array}$$

$\Leftrightarrow$ Every x in S is mapped in the scaled ellipsoid rS:

This is an LMI, aka Semidefinite Program

$$\Leftrightarrow \frac{|Ax|_P}{|x|_P} \leq r$$

$$\Rightarrow \quad \rho \leq r$$

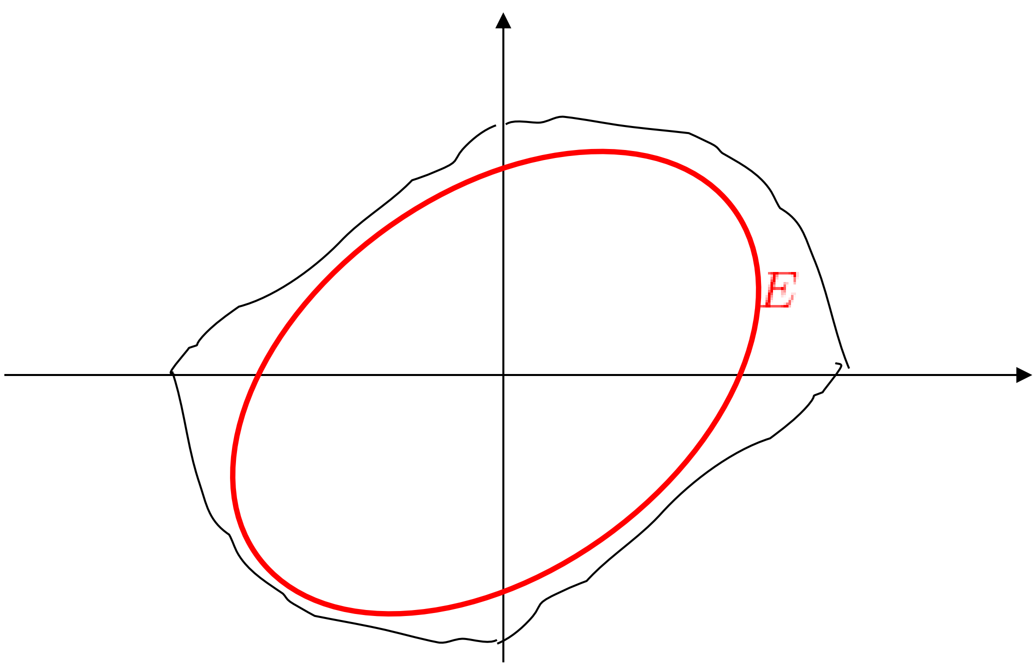


The CQLF method

- Theorem For all $\epsilon > 0$ there exists a norm such that

$$\forall A \in \Sigma, \forall x, |Ax| \leq (\rho + \epsilon)|x|$$ [Rota Strang, 60]

•We can approximate the unit ball of an extremal norm with an ellipsoid



Yet another LMI method

- A strange semidefinite program

$$
\begin{array}{lcl}
\min_{r \in \mathbb{R}^+} & & r \\
\text{s.t.} & & \\
A_1^T P_1 A_1 & \preceq & r^2 P_1, \\
A_2^T P_1 A_2 & \preceq & r^2 P_2, \\
A_1^T P_2 A_1 & \preceq & r^2 P_1, \\
A_2^T P_2 A_2 & \preceq & r^2 P_2, \\
P_i & \succeq & 0.
\end{array}
$$

$$\Rightarrow \quad \rho \leq r$$

[Goebel, Hu, Teel 06]

- But also…

[Daafouz Bernussou 01]

[Bliman Ferrari-Trecate 03]

[Lee and Dullerud 06] …

Yet another LMI method

- An even stranger program:

$$\begin{array}{lcl}
\min_{r \in \mathbb{R}^+} & & r \\
\text{s.t.} & & \\
A_1^T P A_1 & \preceq & r^2 P, \\
(A_2 A_1)^T P (A_2 A_1) & \preceq & r^4 P, \\
(A_2^2)^T P (A_2^2) & \preceq & r^4 P, \\
P & \succeq & 0.
\end{array}$$

$$\rho \leq r$$

[Ahmadi, J., Parrilo, Roozbehani10]

Yet another LMI method

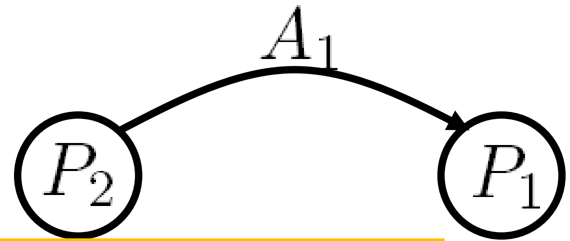
- Questions:
  - Can we characterize all the LMIs that work, in a unified framework?
  - Which LMIs are better than others?
  - How to prove that an LMI works (i.e. is a valid criterion)?
  - Can we provide converse Lyapunov theorems for more methods?

$$\boxed{\frac{1}{\sqrt[2d]{n}}\rho * \leq \rho \leq \rho *}$$

From an LMI to an automaton

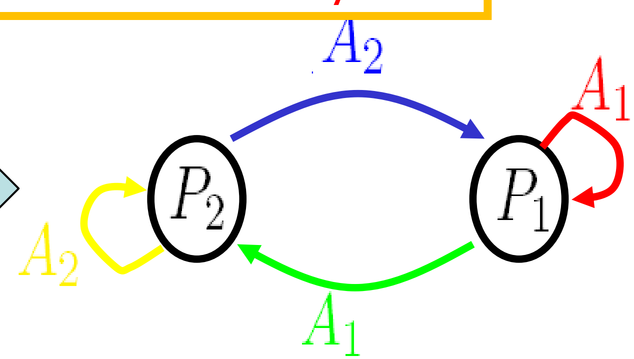
- Automata representation: Given a set of LMIs, construct an automaton like this:

$$A_1^T P_1 A_1 \preceq r^2 P_2$$



> **Theorem:** If G is path-complete, the corresponding semidefinite program is a sufficient condition for stability.

$$\begin{aligned} \min_r \\ \text{s.t.} \\ A_1^T P_1 A_1 &\preceq r^2 P_1, \\ A_2^T P_1 A_2 &\preceq r^2 P_2, \\ A_1^T P_2 A_1 &\preceq r^2 P_1, \\ A_2^T P_2 A_2 &\preceq r^2 P_2, \\ P_i &\succeq 0. \end{aligned}$$

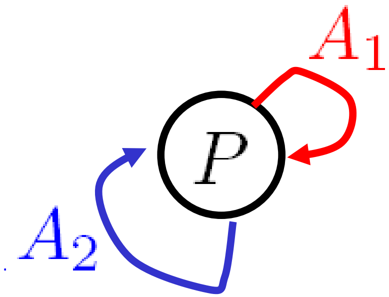


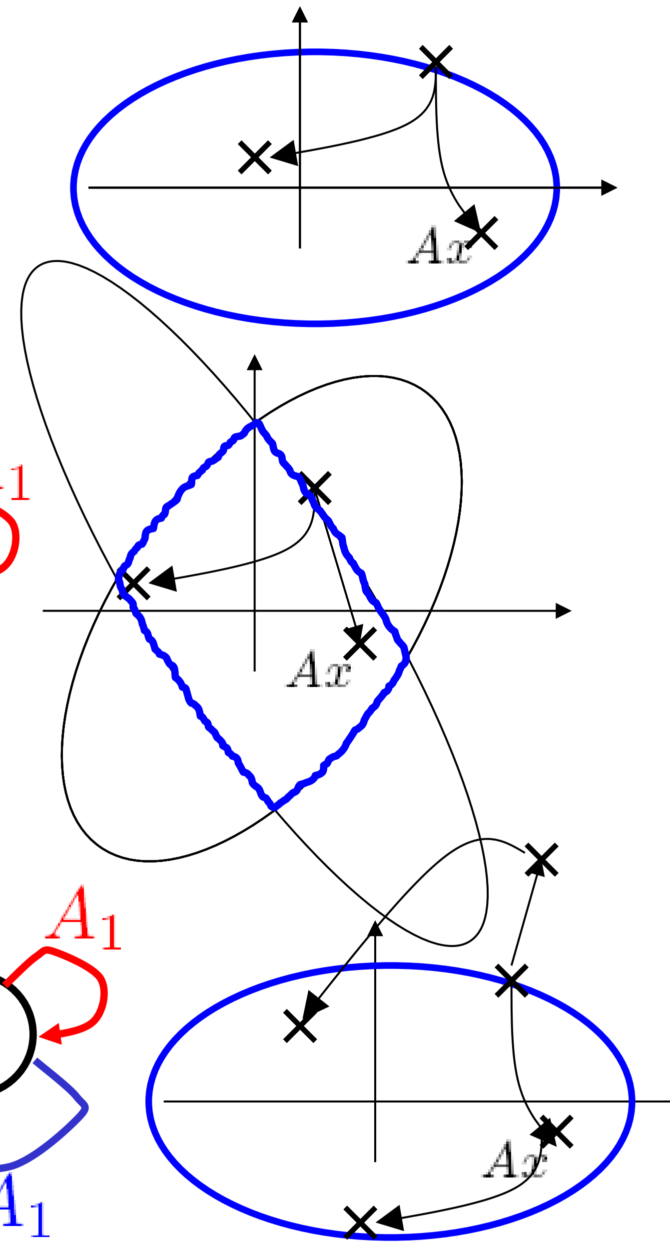
- **Definition:** A labeled graph (with label set A) is path-complete if for any word on the alphabet A, there exists a path in the graph that generates the corresponding word.

[Ahmadi J. Parrilo Roozbehani 14]

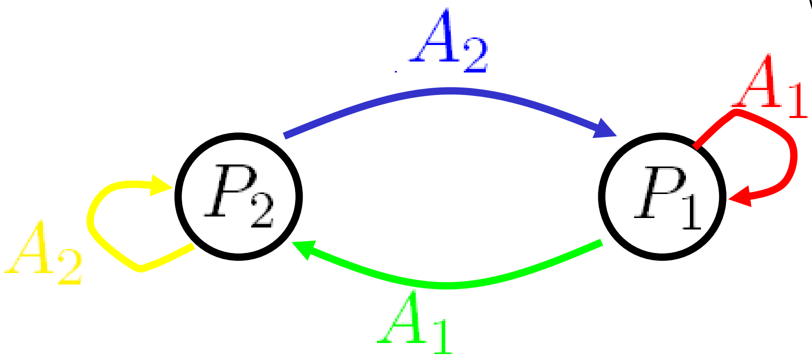
Some examples

- Examples:
  - CQLF



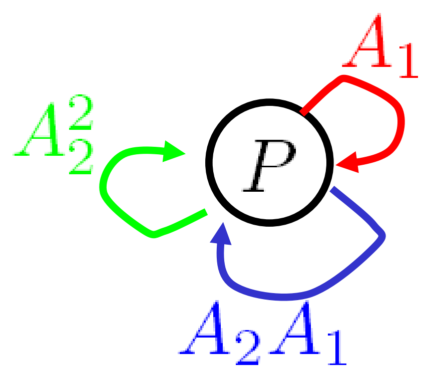


  - Example 1



This type of graph gives a max-of-quadratics Lyapunov function (i.e. intersection of ellipsoids)

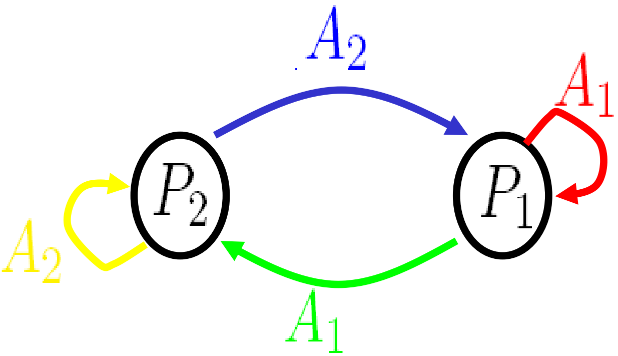
  - Example 2



Invariant set unclear…

An obvious question: are there other valid criteria?

- **Theorem**



$$
\begin{array}{lcl}
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\text{s.t.} & & \\
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P_i & \succeq & 0.
\end{array}
$$

Path complete

???

Sufficient condition for stability

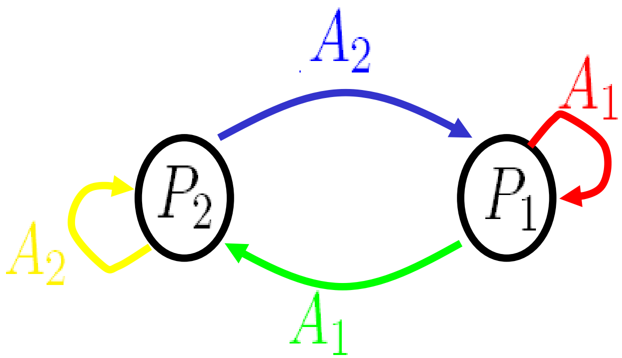
If G is path-complete, the corresponding semidefinite program is a sufficient condition for stability.

- Are all valid sets of equations coming from path-complete graphs?

- …or are there even more valid LMI criteria?

Are there other valid criteria?

- **Theorem:** Non path-complete sets of LMIs are not sufficient conditions for stability [J. Ahmadi Parrilo Roozbehani 17]



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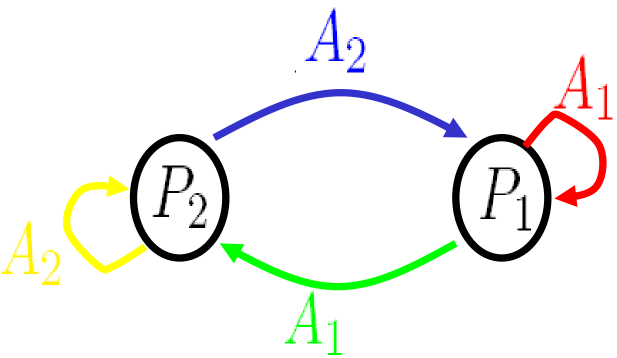
\end{array}
$$

Path complete ??? Sufficient condition for stability

- These results are not limited to LMIs, but apply to other families of Lyapunov inequalities

Are there other valid criteria?

- **Theorem:** Non path-complete sets of LMIs are not sufficient conditions for stability [J. Ahmadi Parrilo Roozbehani 17]



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\begin{array}{lcl}
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A_1^T P_2 A_1 & \preceq & r^2 P_1, \\
A_2^T P_2 A_2 & \preceq & r^2 P_2, \\
P_i & \succeq & 0.
\end{array}
$$

Proof:

- Consider a given graph which is NOT path-complete
- Show that one may construct an ad-hoc set of matrices which satisfies the inequalities (that is, provide the solution)…
- …which however is unstable

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Further results 1: constrained swiching systems

$$\boxed{\sigma = 1001\ldots}$$

Example: Wireless control networks

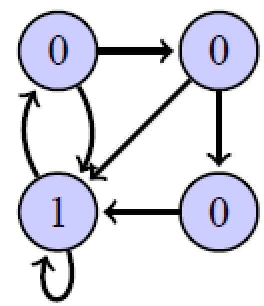
$$\begin{aligned}
\sigma(0) &= 1 & x(1) &= Ax(0) + Bu(0)\\
\sigma(1) &= 0 & x(2) &= A^2x(0) + ABu(0)\\
\sigma(2) &= 0 & x(3) &= A^3x(0) + A^2Bu(0)\\
\sigma(3) &= 1 & x(4) &= A^4x(0) + A^3Bu(0) + Bu(3)
\end{aligned}$$

Often, one can make assumptions on the switching signal

e.g.: The switching signal is constrained by an automaton

Example:

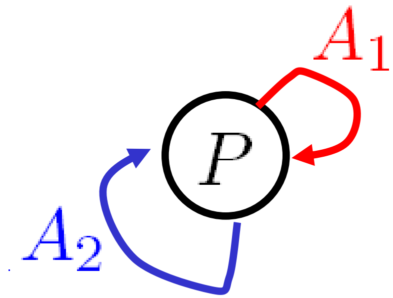
Bounded number of consecutive dropouts (here, 3)

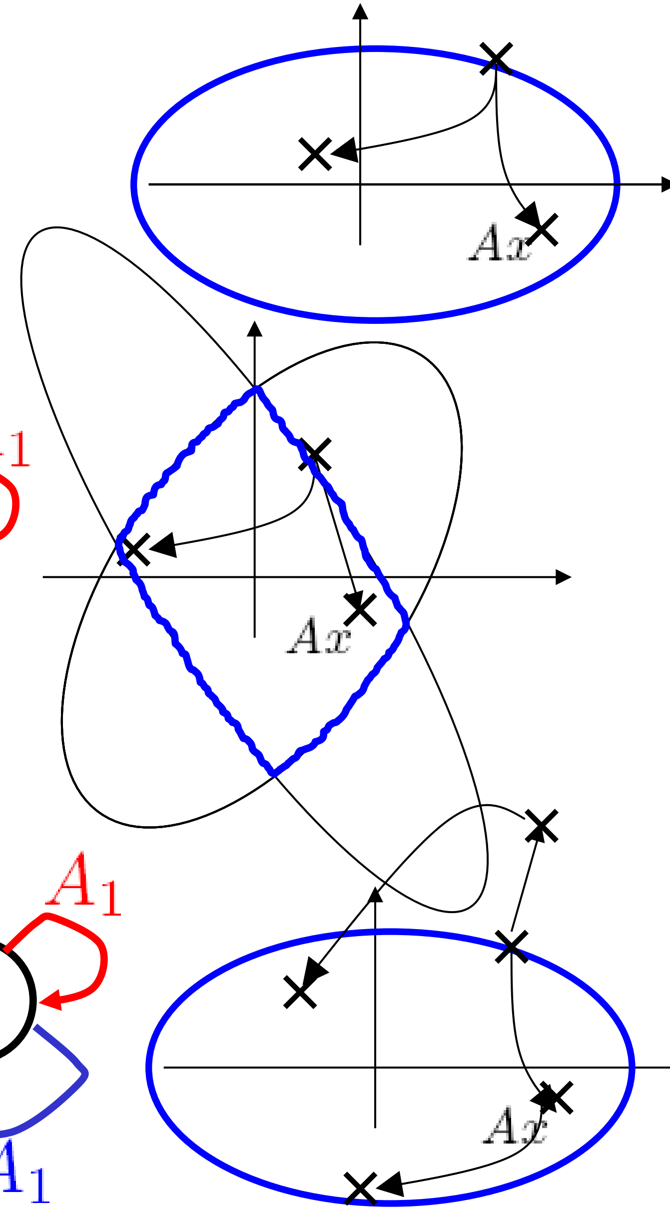


All the results generalize to the constrained case!

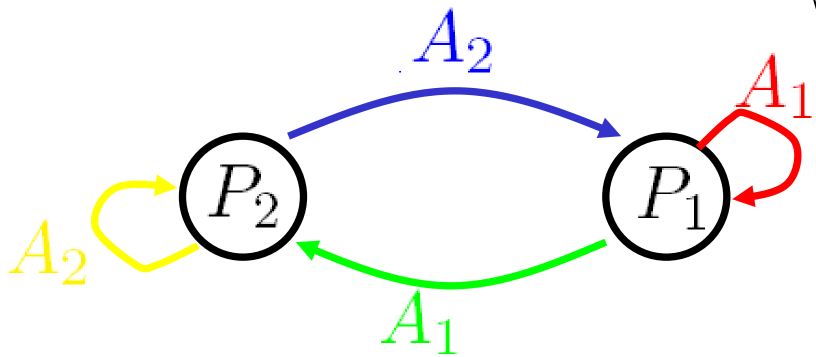
Further results 2: equivalent common Lyapunov Function

- Examples:
  - CQLF



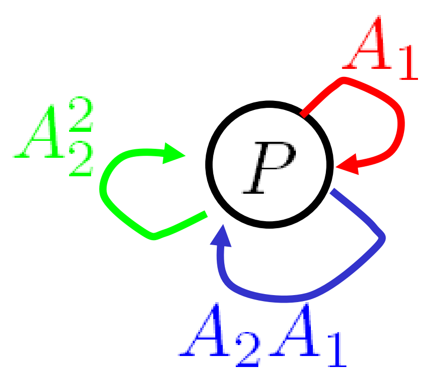


  - Example 1



This type of graph gives a max-of-quadratics Lyapunov function (i.e. intersection of ellipsoids)

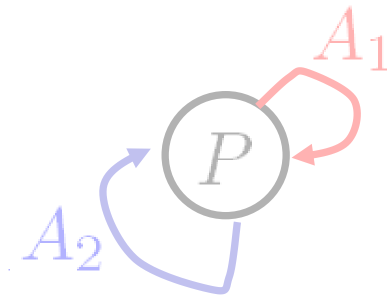
  - Example 2



This type of graph gives a common Lyapunov function for a generating set of words

Further results 2: equivalent common Lyapunov Function

- Examples:
  - CQLF



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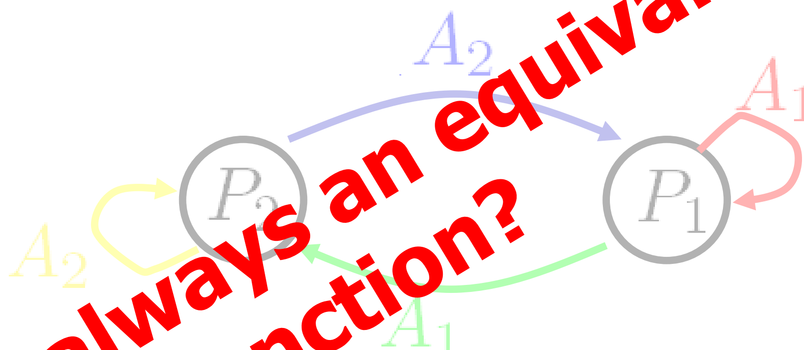

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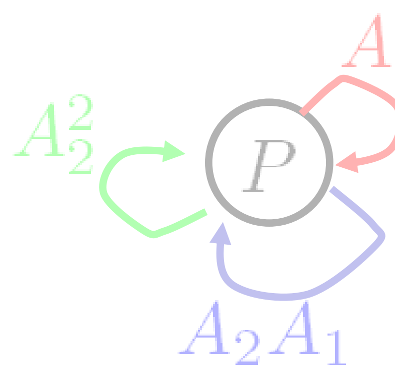

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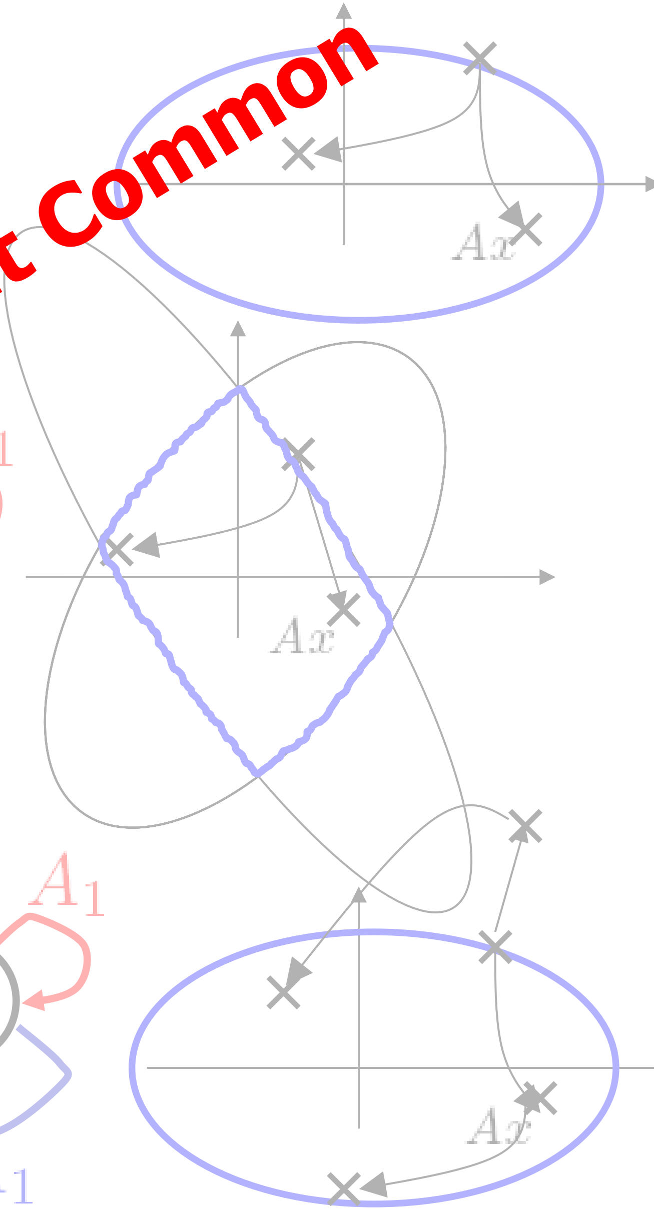


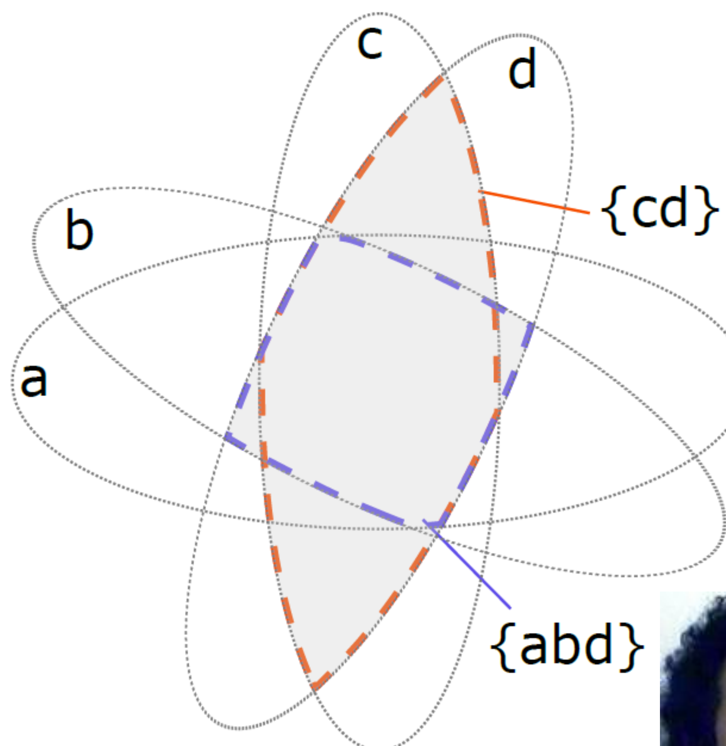
Is there always an equivalent Common Lyapunov Function?

Further results 2: equivalent common Lyapunov Function

- Theorem Every path-complete criterion implies the existence of a Common Lyapunov function. This Lyapunov function can be expressed analytically as the minimum of maxima of the quadratic functions.

[Angeli Athanasopoulos Philippe J., 2017]

$$
\begin{aligned}
&\min_{r\in\mathbb{R}^+} \\
&\text{s.t.} \\
&A_1^T P_1 A_1 && \preceq \\
&A_2^T P_1 A_2 && \preceq \\
&A_1^T P_2 A_1 && \preceq \\
&A_2^T P_2 A_2 && \preceq \\
&P_i && \succeq
\end{aligned}
$$



$$\min_{1,\dots,S_k\subseteq\{1,\dots,P\}} \max_{s\in S_i} V_s(x)$$

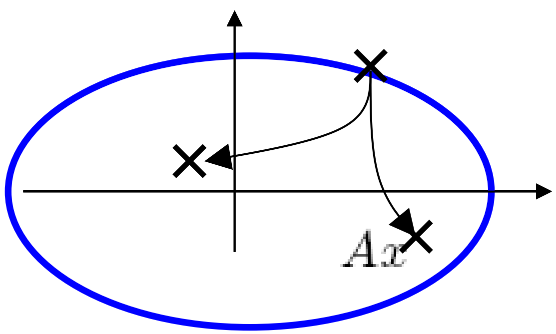
Is a Common Lyapunov function for some sets Si

- Proof based on the observer automaton

(cfr Cassandras-Lafortune)

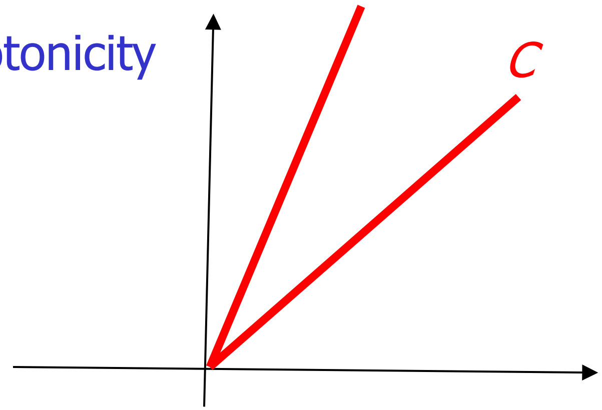
Further generalizations (3)

Stability



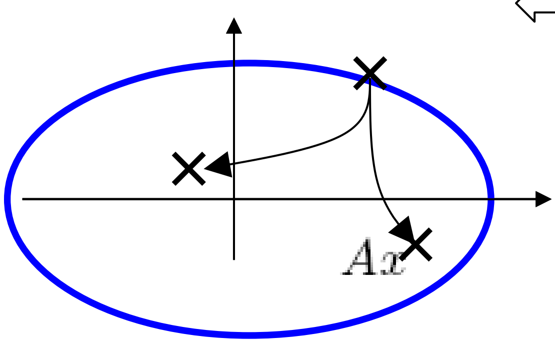
$\Longleftrightarrow$

Monotonicity



Joint work with F. Forni, R. Sepulchre

Stability



$\Longleftrightarrow$

Controller design

$$x(t+1) = \begin{cases} Ax(t) + \boxed{bu(t)} & \text{if } \sigma(t) = 1, \\ Ax(t), & \text{if } \sigma(t) = 0 \end{cases}$$

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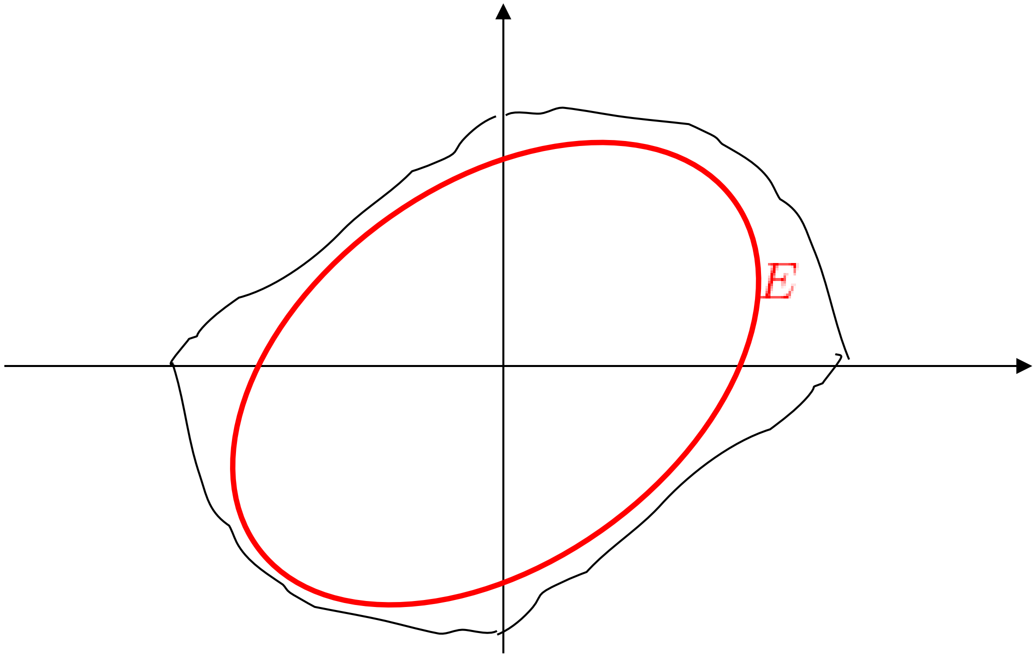
SAMSA

Series, Automata, Matrices, Symbolic dynamics, and their Applications

- Theorem For all $\epsilon > 0$ there exists a norm such that

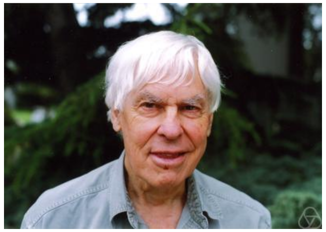
$$\forall A \in \Sigma, \forall x, |Ax| \leq (\rho + \epsilon)|x|$$ [Rota Strang, 60]

- We can approximate the unit ball of an extremal norm with an ellipsoid

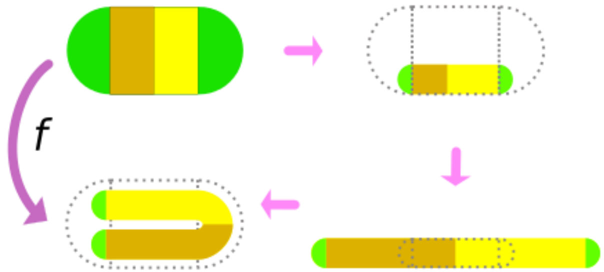


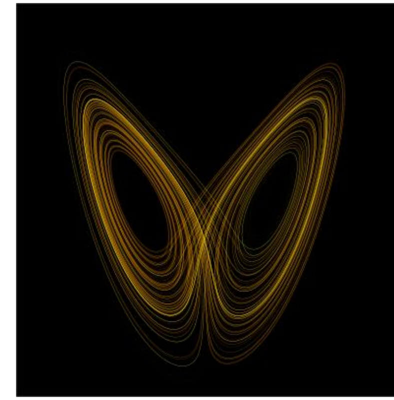
Standing on Giants shoulders

Symbolic dynamics



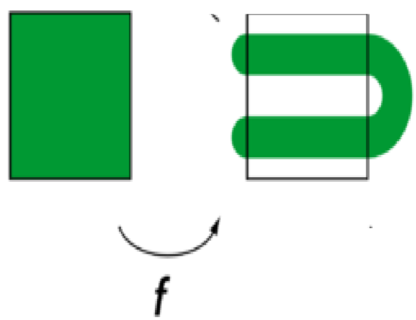
Stephen Smale
1930-
Fields Medsl 1966
Wolf prize 2007



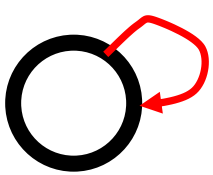


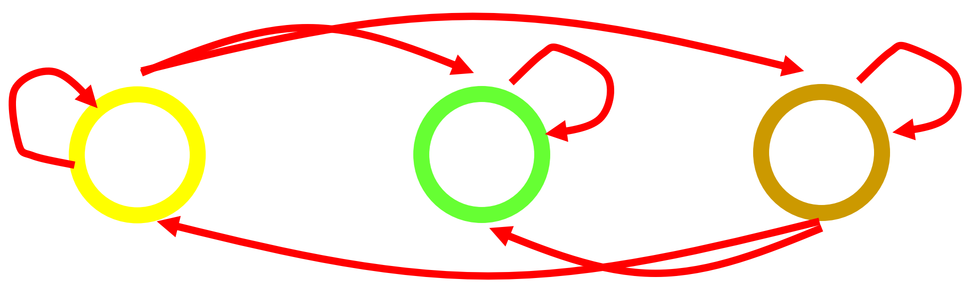
Gustav Hedlund
1904-1993

Marston Morse
1892-1977



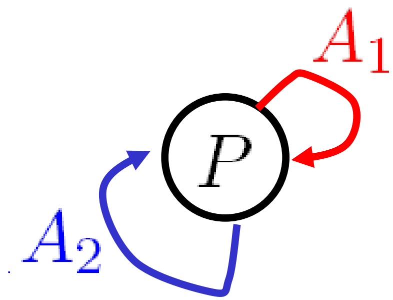
Marston Morse et Gustav A. Hedlund (1938), « Symbolic dynamics », *Amer. J. Math*, vol. 60, n^{o} 4, 1938



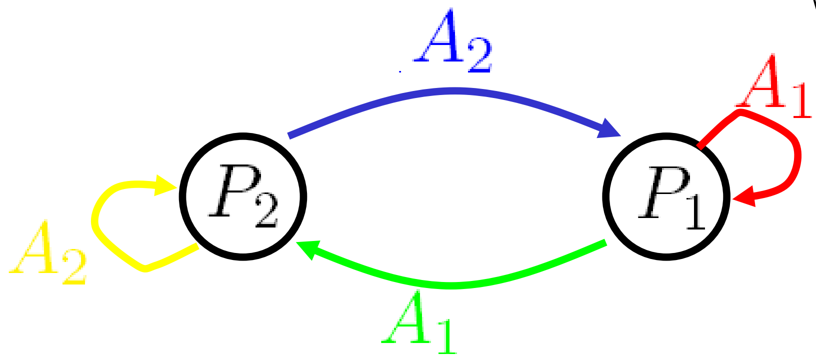


Path-complete methods are 'algorithmic symbolic dynamics'

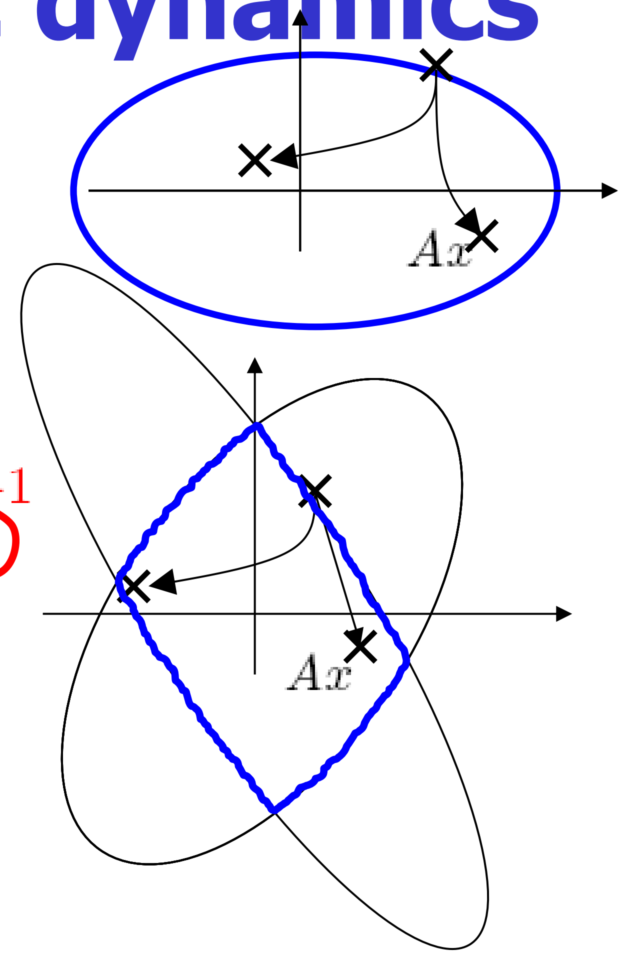
- Examples:
  - CQLF



  - Example 1

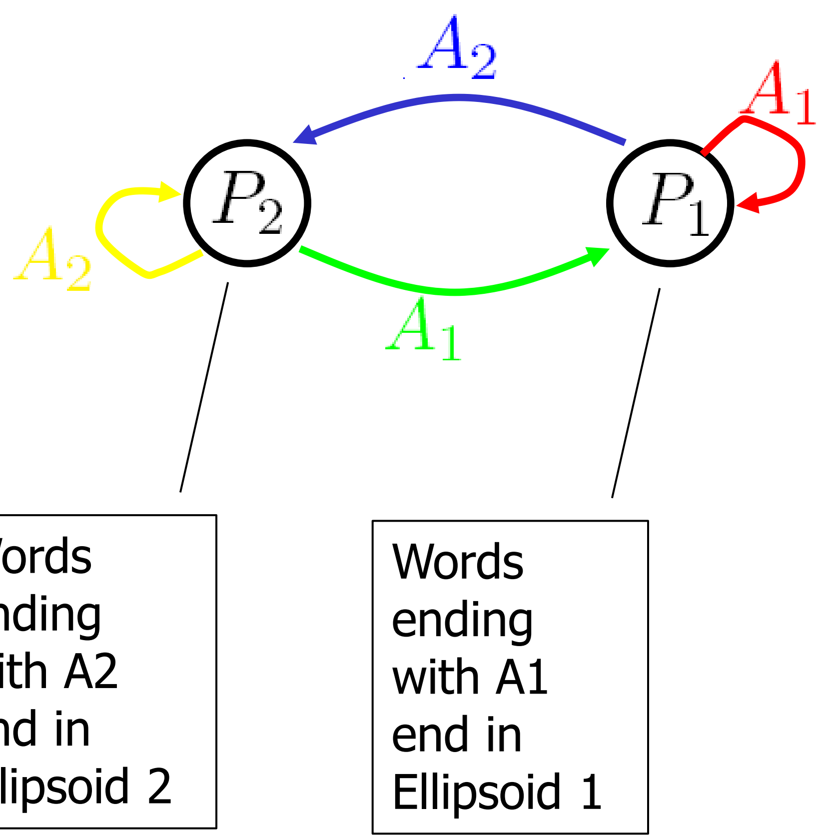


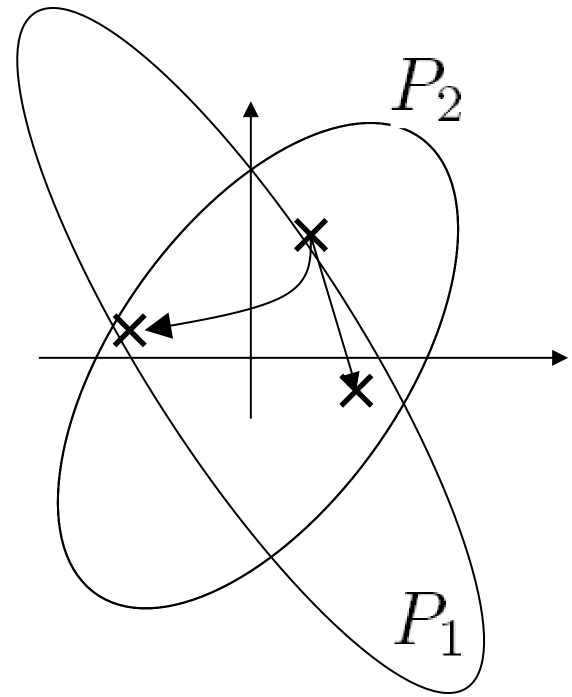
This type of graph gives a max-of-quadratics Lyapunov function (i.e. intersection of ellipsoids)



Path-complete methods are ‘algorithmic symbolic dynamics’

- Path-complete Lyapunov Functions provide a ‘covering’ of the whole set of possible dynamics
  - Example 1

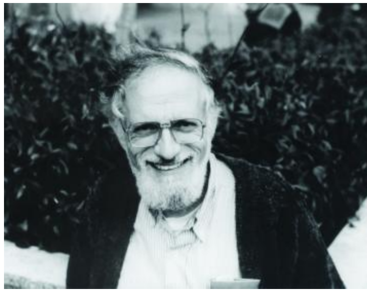




Outline

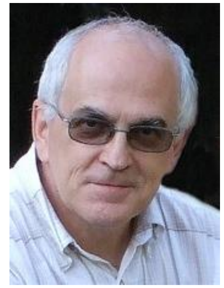
- Switching systems
- Path-complete methods for switching systems stability
- Further results
- SAMSA? ‘Series, Automata, Matrices, **Symbolic dynamics**, and their Applications’
- **Conclusion and open questions**

Conclusion: a perspective on switching systems




[Furstenberg Kesten, 1960]

[Gurvits, 1995]



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[Parrilo Jadbabaie 20

60s 70s

Mathematical properties

90s

TCS inspired
Negative
Complexity results

2000s

Lyapunov/LMI Techniques (S-procedure)

now

CPS applications ⇔ Smart algorithms

Thanks!

Questions?

Ads

[The JSR Toolbox: http://www.mathworks.com/matlabcentral/fileexchange/33202-the-jsr-toolbox](http://www.mathworks.com/matlabcentral/fileexchange/33202-the-jsr-toolbox)
[Van Keerberghen, Hendrickx, J. HSCC 2014]
The CSS toolbox, 2015

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References:
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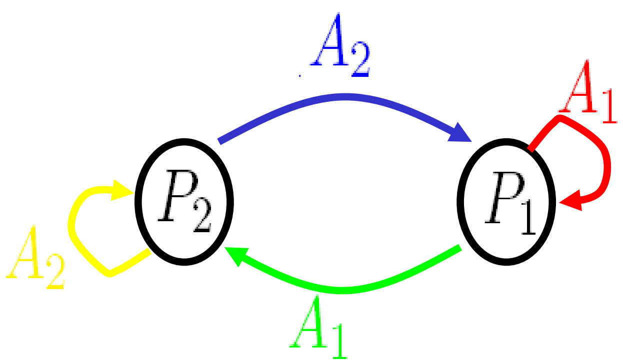
Further results and open problems

This approach naturally generalizes to other problems



$$
\begin{array}{lll}
\min_{r \in \mathbb{R}+} & & r \\
\text{s.t.} & & \\
A_1^T P_1 A_1 & \preceq & r^2 P_1, \\
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\end{array}
$$

Optimize on optimization problems!

This framework is generalizable to harder problems

- **Constrained switching systems**
- Path-complete monotonicity
- Automatically optimized abstractions of cyber-physical systems
- …

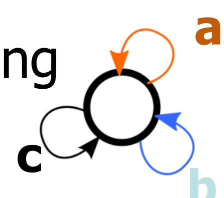
Constrained switching sequences

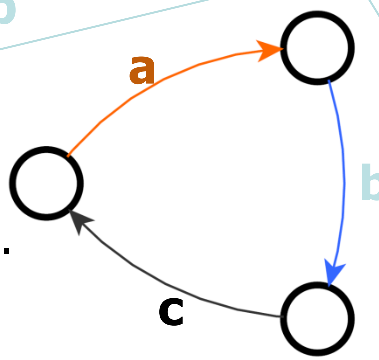
Switching sequences on regular languages

$G(V,E)$ Directed & Labeled $\quad e=(v_i,v_j,k)\in E \quad k\in\{1,\cdots,N\}$

$\sigma(1),\sigma(2),\cdots$ **admissible** if $\exists p=\{(v_i,v_j,\sigma(1)),\,(v_j,v_\ell,\sigma(2)),\,\cdots\}$

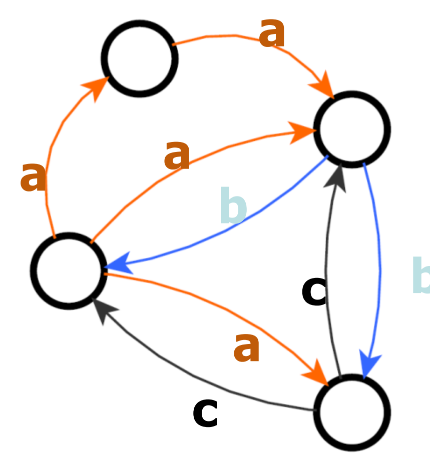
everything





... abcabcabc ...

... ~~ac~~ ...



... ~~bb~~ ...

... ~~cc~~ ...

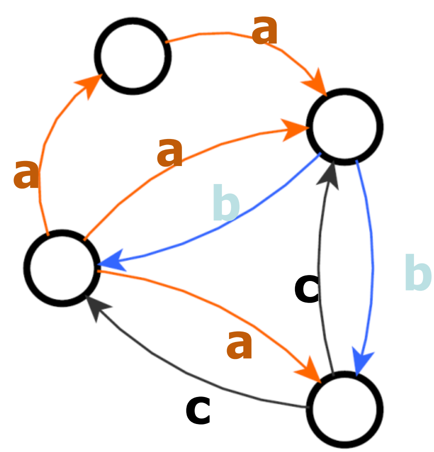
... aab

...

Constrained switching sequences

$G(V, E)$ Directed & Labeled $\quad e = (v_i, v_j, k) \in E \quad k \in \{1, \cdots, N\}$

$\sigma(1), \sigma(2), \cdots$ **admissible** if $\exists p = \{(v_i, v_j, \sigma(1)), (v_j, v_\ell, \sigma(2)), \cdots\}$



Stability

$$\lim_{t\to\infty} x_t = \lim_{t\to\infty} A_{\sigma(t-1)} \cdot \ldots \cdot A_{\sigma(0)} x_0 = 0$$
$$\forall x_0 \in \mathbb{R}^n, \forall \sigma(0), \sigma(1), \cdots \in G$$

Again, one can define a **Constrained Joint Spectral Radius** (CJSR), as the asymptotic worse-case rate of growth of x(t)

Application:
Systems with intermittent dropouts

$$\boxed{\sigma = 1001\ldots}$$

Example: Wireless control networks

$\sigma(0) = 1$ $\qquad x(1) = Ax(0) + Bu(0)$

$\sigma(1) = 0$ $\qquad x(2) = A^2x(0) + ABu(0)$

$\sigma(2) = 0$ $\qquad x(3) = A^3x(0) + A^2Bu(0)$

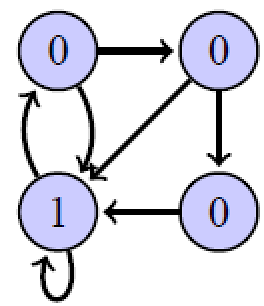
$\sigma(3) = 1$ $\qquad x(4) = A^4x(0) + A^3Bu(0) + Bu(3)$

Often, one can make assumptions on the switching signal

e.g.: The switching signal is constrained by an automaton

Example:

Bounded number of consecutive dropouts (here, 3)



Constrained switching and multinorms

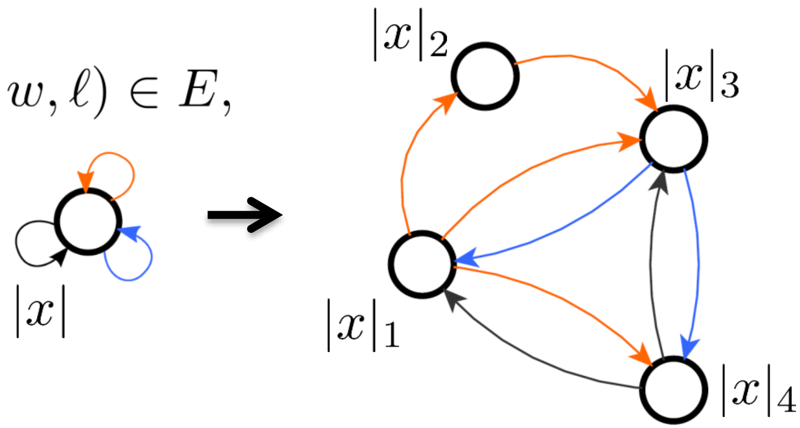
- **Problem:** stability does not imply the existence of a contractive norm (no converse Lyapunov theorem)!

Constrained switching and multinorms

- CJSR as an infimum over **sets** of norms

$$\rho(G(V,E),M) = \inf_{\gamma} \; s.t.\; \exists\{Q_v \succ 0,\, v \in V\} : \forall (v,w,\ell) \in E, \quad A_\ell^\top Q_w A_\ell \preceq \gamma^2 Q_v$$

$$\rho^* = \inf_{|\cdot|_1,\cdots,|\cdot|_{|V|}} \left\{ \begin{array}{l} |A_k x|_j \leq \gamma |x|_i, \\ \forall (v_i, v_j, k) \in E,\ \forall x \in \mathbb{R}^n \end{array} \right.$$



Theorem:

$$\boxed{\frac{1}{\sqrt{n}}\rho^* \leq \rho \leq \rho^*}$$

Corollary: PTAS

[Philippe, Essick, Dullerud, J. 2016]

An interesting bound

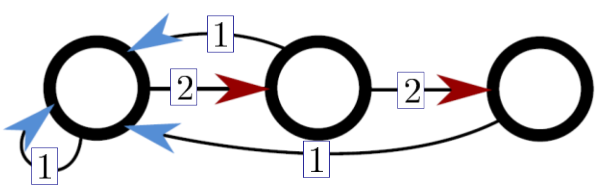
Theorem: Consider the LMI bound on the cjsr

$$\rho^* = \begin{cases} \inf \gamma \\ s.t.\ \exists \{Q_v \succ 0,\ v \in V\} : \forall (v,w,\ell) \in E, \\ A_\ell^\top Q_w A_\ell \preceq \gamma^2 Q_v \end{cases}$$

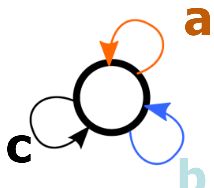
$$\boxed{\frac{1}{\sqrt{r}}\rho^* \leq \rho \leq \rho^*}$$

where r is the spectral radius of the adjacency matrix of the automaton

[Legat J. Parillo, HSCC 2016]



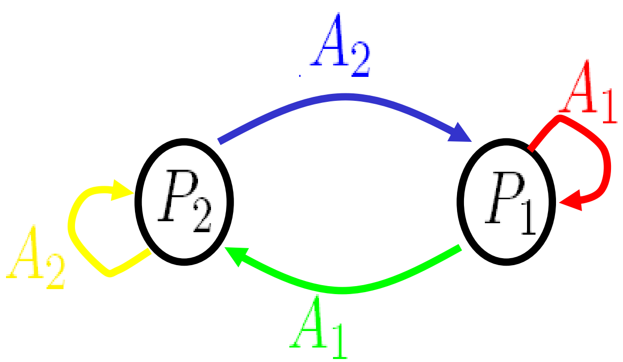
$$\boxed{\frac{1}{\sqrt{1.84}}\rho^* \leq \rho \leq \rho^*}$$



$$\boxed{\frac{1}{\sqrt{m}}\rho^* \leq \rho \leq \rho^*}$$

Further results and open problems

This approach naturally generalizes to other problems



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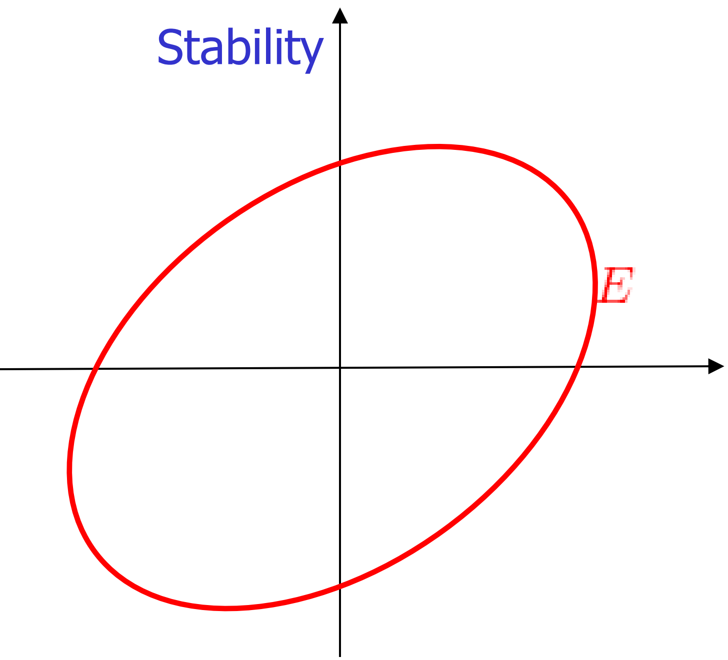
Optimize on optimization problems!

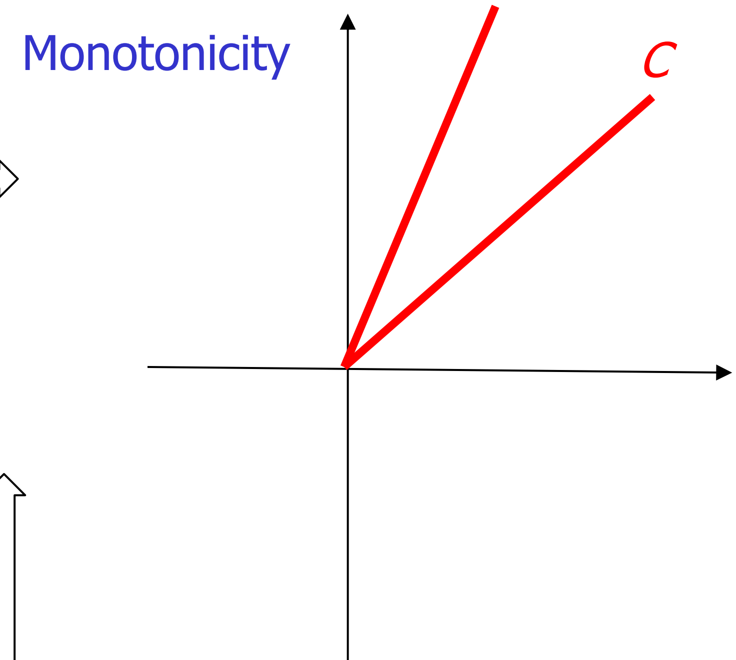
This framework is generalizable to harder problems

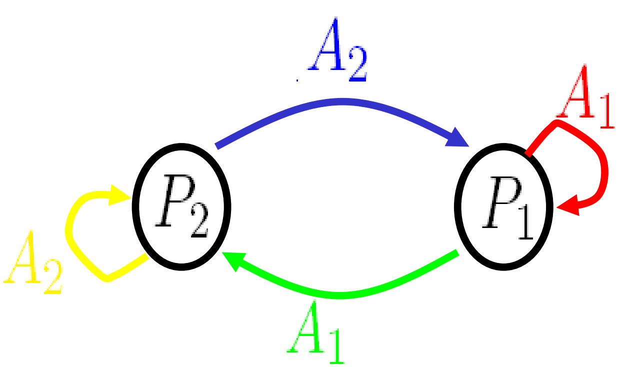
- Constrained switching systems
- **Path-complete monotonicity**
- Automatically optimized abstractions of cyber-physical systems
- …

Path-complete monotonicity

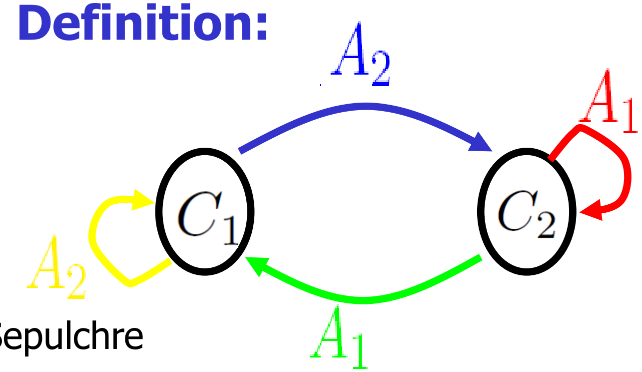
Replace invariant compact sets by invariant cones:







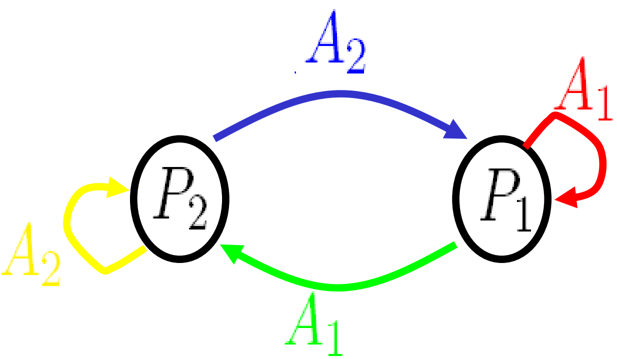
Definition:



Joint work with F. Forni, R. Sepulchre

Further results and open problems

This approach naturally generalizes to other problems



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Optimize on optimization problems!

This framework is generalizable to harder problems

- Constrained switching systems
- Path-complete monotonicity
- **Automatically optimized abstractions of cyber-physical systems**
- …

Automatically optimized abstractions of cyber-physical systems

- Theorem Every path-complete criterion implies the existence of a Common Lyapunov function. This Lyapunov function can be expressed analytically as the minimum of maxima of the quadratic functions.

[Angeli Athanasopoulos Philippe J., 2017]

$$
\begin{array}{ll}
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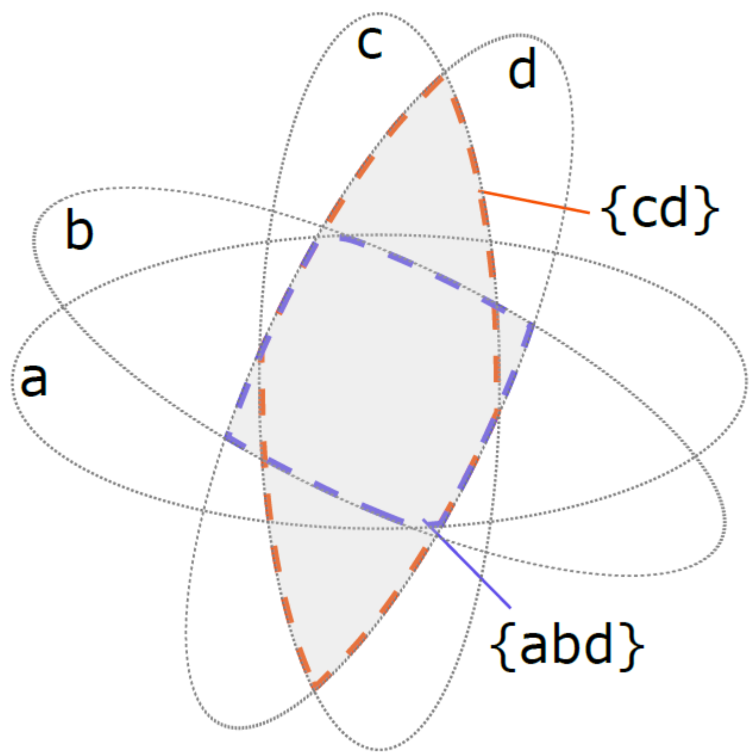
P_i & \succeq
\end{array}
$$



$$\min_{1,\dots,S_k \subseteq \{1,\dots,P\}} \max_{s \in S_i} V_s(x)$$

Is a Common Lyapunov function for some sets Si

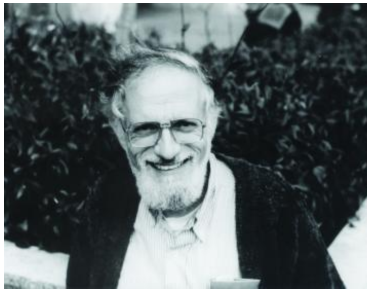
- Proof based on the observer automaton

(cfr Cassandras-Lafortune)

Outline

- Joint spectral characteristics
- Path-complete methods for switching systems stability
- Further results and open problems
- **Conclusion and perspectives**

Conclusion: a perspective on switching systems

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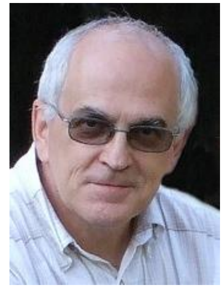

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[Lee Dullerud 2006]

[Parrilo Jadbabaie 2008]

(sensor) networks

Wireless control

Bisimulation design

consensus problems

Social/big data control

…

60s 70s

Mathematical properties

90s

TCS inspired
Negative
Complexity results

2000s

Lyapunov/LMI
Techniques
(S-procedure)

now

CPS applic.
Ad hoc techniques

Thanks!

Questions?

Ads

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Questions?

References:

- R. M. Jungers, A. D'Innocenzo and M. D. Di Benedetto. **Modeling, analysis, and design of linear systems with switching delays.** IEEE TAC, 2015.
- R. M. Jungers, A. D'Innocenzo and M. D. Di Benedetto. **Further results on controllability of linear systems with switching delays.** *Proc. of IFAC WC 2014.*
- R. M. Jungers, A. D'Innocenzo and M. D. Di Benedetto. **How to control linear systems with switching delays?** *Proc. of ECC 2014..*
- R. M. Jungers, A. D'Innocenzo and M. D. Di Benedetto. **Feedback stabilization of dynamical systems with switched delays.** *Proceedings of the IEEE Conference on Decision and Control 2012, Hawai, 2012.*
- R. M. Jungers, M. Heemels. **Controllability of linear systems suject to packet losses**. *Proc. Of ADHS, Atlanta, 2015*.
- R. M. Jungers, A. Kundu, and M. Heemels*.* **Exact characterization of observability and controllability with packet losses.** *Proc. of Allerton 2015*.

… these and more on
http://perso.uclouvain.be/raphael.jungers/